CHAPTER 5

# Logarithm

## Exercise

1. If $\log_3 x \,.\, \log_x 2x \,.\, \log_{2x} y = \log_x (x^2)$, then $y$ equals to
   (a) $\frac{9}{2}$ (b) 9
   (c) 18 (d) 27

2. The value of $\frac{\log_a n}{\log_{ab} n}$ is
   (a) $1 + \log_a b$
   (b) $1 + \log_b a$
   (c) Both (a) and (b)
   (d) None of these

3. If $\log_{10} m = b - \log_{10} n$, then $m$ is
   (a) $\frac{b}{m}$ (b) $10^b n$
   (c) $b - 10^n$ (d) $\frac{10^b}{n}$

4. If $\log_b a \times \log_y x = \log_c a$, then the values of $x$ and $y$ are respectively
   (a) $b$ and $c$ (b) $c$ and $b$
   (c) $c$ and $a$ (d) $b$ and $a$

5. If $(\log_3 x)^2 + \log_3 x < 2$, then which one of the following is correct?
   (a) $0 < x < \frac{1}{9}$ (b) $\frac{1}{9} < x < 3$
   (c) $3 < x < 0$ (d) $\frac{1}{9} \le x \le 3$

6. $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$ is equal to
   (a) 1 (b) 2
   (c) 3 (d) 4

7. The solution of $2 \log (x + 1) - \log (x^2 - 1) = \log 2$ is
   (a) 1 (b) –1
   (c) 0 (d) 3

8. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then find the value of $a^a \,.\, b^b \,.\, c^c$ is
   (a) 1 (b) 2
   (c) –1 (d) 0

9. If $\log \frac{75}{35} + 2\log \frac{7}{5} - \log \frac{105}{x} - \log \frac{13}{25} = 0$, then find the value of $x$.
   (a) 90 (b) 60
   (c) 13 (d) 45

10. If $x + \log_{15} (1 + 3^x) = x \log_{15} 5 + \log_{15} 2$, where $x$ is an integer then what is $x$ equal to?
    (a) – 3 (b) 2
    (c) 1 (d) 3

**ANSWERS**

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| **1.** | (b) | **2.** | (a) | **3.** | (d) | **4.** | (a) | **5.** | (b) | **6.** | (b) | **7.** | (d) | **8.** | (a) | **9.** | (c) | **10.** | (c) |

# Explanations

1. (b) $\log_3 x.\log_x 2x.\log_{2x} y = \log_x x^2$
$\Rightarrow \log_3 y = 2\log_x x \quad \{\because \log_a b \times \log_b c = \log_a c\}$
$\Rightarrow \log_3 y = 2$
$\Rightarrow y = 3^2 = 9 \quad \{\because \log_a b = c \Rightarrow b = a^c\}$

2. (a) $\dfrac{\log_a n}{\log_{ab} n} = \dfrac{\log n \times \log ab}{\log a \times \log n} = \dfrac{\log a + \log b}{\log a}$
$= 1 + \dfrac{\log b}{\log a} = 1 + \log_a b$

3. (d) $\log_{10} m = b - \log_{10} n$
$\Rightarrow b = \log_{10} m + \log_{10} n \Rightarrow b = \log_{10} mn$
$\Rightarrow mn = 10^b \Rightarrow m = \dfrac{10^b}{n}$

4. (a) Given $\log_b a \times \log_y x = \log_c a$
$\Rightarrow \dfrac{\log a}{\log b} \times \dfrac{\log x}{\log y} = \dfrac{\log a}{\log c}$
$\Rightarrow \dfrac{\log x}{\log y} = \dfrac{\log b}{\log c}$
$\Rightarrow x = b$ and $y = c$

5. (b) $(\log_3 x)^2 + \log_3 x < 2$
Let $\log_3 x = y$
$\Rightarrow y^2 + y - 2 < 0 \Rightarrow (y+2)(y-1) < 0$
$\Rightarrow -2 < y < 1 \Rightarrow -2 < \log_3 x < 1$
$\Rightarrow 3^{-2} < x < 3^1$
$\Rightarrow \dfrac{1}{9} < x < 3$

6. (b) $\dfrac{1}{\log_{xy}(xyz)} + \dfrac{1}{\log_{yz}(xyz)} + \dfrac{1}{\log_{zx}(xyz)}$
$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$
$= \log_{xyz}(xy)(yz)(zx)$
$= \log_{xyz}(xyz)^2 = 2\log_{xyz}(xyz) = 2$

7. (d) $2\log(x+1) - \log(x^2-1) = \log 2$
$\Rightarrow \log\dfrac{(x+1)^2}{(x^2-1)} = \log 2$
$\Rightarrow \dfrac{x+1}{x-1} = 2$
$\Rightarrow 2x - 2 = x + 1 \Rightarrow x = 3$

8. (a) Let $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b} = k$
$\Rightarrow \log a = (b-c)k, \log b = (c-a)k, \log c = (a-b)k$
$\Rightarrow a = e^{(b-c)k}, b = e^{(c-a)k}, c = e^{(a-b)k}$
So, $a^a \,.\, b^b \,.\, c^c$
$= e^{a(b-c)k} \,.\, e^{b(c-a)k} \,.\, e^{c(a-b)k}$
$= e^{(ab-ac+bc-ab+ca-bc)k}$
$= e^0 = 1$

9. (c) $\log\dfrac{75}{35} + 2\log\dfrac{7}{5} - \log\dfrac{105}{x} - \log\dfrac{13}{25} = 0$
$\Rightarrow \log\dfrac{75}{35} + \log\dfrac{49}{25} + \log\dfrac{x}{105} + \log\dfrac{25}{13} = 0$
$\Rightarrow \log\dfrac{75}{35} \times \dfrac{49}{25} \times \dfrac{x}{105} \times \dfrac{25}{13} = 0$
$\Rightarrow \dfrac{75}{35} \times \dfrac{49}{25} \times \dfrac{x}{105} \times \dfrac{25}{13} = 1 \Rightarrow x = 13$

10. (c) $x + \log_{15}(1+3^x) = x\log_{15} 5 + \log_{15} 12$
$\Rightarrow \log_{15} 15^x + \log_{15}(1+3^x) = \log_{15} 5^x + \log_{15} 12$
$\Rightarrow 15^x(1+3^x) = 5^x(12)$
$\Rightarrow 3^x(1+3^x) = 12$
$\Rightarrow (3^x)^2 + (3^x) - 12 = 0$
$\Rightarrow (3^x+4)(3^x-3) = 0$
$\Rightarrow 3^x = -4$ or $3^x = 3 \quad \{\because 3^x$ can never be –ve$\}$
$\Rightarrow x = 1 \quad \{\because x$ is an integer$\}$